Axino as a sterile neutrino and *R* parity violation

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We suggest that the axino can be a natural candidate for a sterile neutrino which would accommodate the LSND data with atmospheric and solar neutrino oscillations. It is shown that the so-called $3+1$ scheme can be easily realized when supersymmetry breaking is mediated by gauge interactions and also *R* parity is properly broken. Among the currently possible solutions to the solar neutrino problem, only the large angle MSW oscillation is allowed in this scheme. The weak scale value of the Higgs μ parameter and the required size of *R*-parity violation can be understood by means of spontaneously broken Peccei-Quinn symmetry.

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Current data from the atmospheric and solar neutrino experiments are beautifully explained by oscillations among three active neutrino species [1]. Additional data in favor of neutrino oscillation have been obtained in the Liquid Scintillation Neutrino Detector (LSND) experiment [2]. Reconciliation of these experimental results requires three distinct mass-squared differences, implying the existence of a sterile neutrino ν_s. In the four-neutrino oscillation framework, there are two possible scenarios [3,4]: the $2+2$ scheme in which two pairs of close mass eigenstates are separated by the LSND mass gap ~ 1 eV and the $3+1$ scheme in which one mass is isolated from the other three by the LSND mass gap. It has been claimed that the LSND results can be compatible with various short-base-line experiments only in the context of the $2+2$ scheme [5]. However, according to the new LSND results [6], the allowed parameter regions are shifted to a smaller mixing angle, thereby allowing the $3+1$ scheme to be phenomenologically viable [3,4,7]. Although it can be realized in a rather limited parameter space, the $3+1$ scheme is attractive since the fourth (sterile) neutrino can be added without changing the most favorable picture that the atmospheric and solar neutrino data are explained by the predominant $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\mu, \nu_\tau$ oscillations, respectively. In particular, the $3+1$ scheme with the heaviest ν_s would be an interesting explanation of all existing neutrino data. On the other hand, it is rather difficult to find a well-motivated particle physics model which would yield the desired four-neutrino masses and mixing in a consistent way [8].

In this paper, we show that the $3+1$ scheme can be easily realized in a supersymmetric model with U(1) Peccei-Quinn (PQ) symmetry when supersymmetry (SUSY) breaking is mediated by gauge interactions. In such model, the axino can be as light as 1 eV, and so play the role of a sterile neutrino [9]. A proper axino-neutrino mixing can be induced by *R*-parity-violating couplings which appear as a consequence of the spontaneous breaking of $\mathrm{U}(1)_{PQ}$. It turns out that only the large angle Mikheyev-Smirnov-Wolfenstein (MSW) solution to the solar neutrino problem is allowed in our model. The weak scale value of the Higgs μ parameter and the required size of *R*-parity violation can be understood by means of the Frogatt-Nielsen mechanism [10] of spontaneously broken $\mathrm{U}(1)_{PQ}$.

The model under consideration contains three sectors: the observable sector, the SUSY-breaking sector, and the PQ sector. The observable sector contains the usual quarks, leptons, and two Higgs superfields, i.e., the superfields of the minimal supersymmetric standard model (MSSM). The SUSY-breaking sector contains a gauge-singlet Goldstino superfield X and the gauge-charged messenger superfields Y, Y^c as in conventional gauge-mediated SUSY-breaking models [11]. Finally the PQ sector contains gauge-singlet superfields S_k $(k=1,2,3)$ which break $\mathrm{U}(1)_{PQ}$ by their vacuum expectation values (VEVs), as well as gauge-charged superfields T, T^c which have Yukawa coupling with some of S_k.

The Kähler potential of the model can always be written as

$$K = \sum_I \Phi_I^\dagger \Phi_I + \cdots, \tag{1}$$

where Φ_I denote generic chiral superfields of the model and the ellipsis stands for (irrelevant) higher-dimensional operators which are suppressed by some powers of $1/M_*$ where M_* corresponds to the cutoff scale of the model. Throughout this paper, we assume that M_* corresponds to the gauge coupling unification scale, so $M_* \approx 10^{16}$ GeV. The superpotential of the model is given by

$$W = hS_3(S_1S_2 - f_{PQ}^2) + \kappa S_1 T T^c + \lambda X Y Y^c + W_{\mathrm{MSSM}} + W_{\mathrm{SB}}, \tag{2}$$

where W_{MSSM} involves the MSSM fields, and W_{SB} describes SUSY-breaking dynamics enforcing that X develop a SUSY-breaking VEV:

$$\lambda X = M_X + \theta^2 F_X.$$

TABLE I. Peccei-Quinn charges of superfields. All superfields in the SUSY-breaking sector are assumed to be $U(1)_{PQ}$ neutral.

Fields	Peccei-Quinn charge
PQ sector	
S_1	-1
S_2	1
S_3	0
T	0
T^c	1
MSSM sector	
H_1	1
H_2	1
L	2
E^c	-3
Q	0
U^c	-1
D^c	-1

This VEV generates soft masses of the MSSM fields [11],

$$m_{\rm soft} \approx \frac{\alpha}{2\pi}\frac{F_X}{M_X},$$

which are assumed to be of order the weak scale. One can easily arrange the symmetries of the model, e.g., $U(1)_{PQ}$ and an additional discrete symmetry, to ensure that $W_{\rm MSSM}$ is given by

$$\begin{aligned} W_{\rm MSSM} = & y_{ij}^{(E)} H_1 L_i E_j^c + y_{ij}^{(D)} H_1 Q_i D_j^c + y_{ij}^{(U)} H_2 Q_i U_j^c \\ & + \frac{y_0}{M_*} S_1^2 H_1 H_2 + \frac{y_i'}{M_*^2} S_1^3 L_i H_2 + \frac{\gamma_{ijk}}{M_*} S_1 L_i L_j E_k^c \\ & + \frac{\gamma'_{ijk}}{M_*} S_1 L_i Q_j D_k^c + \cdots, \end{aligned} \tag{3}$$

where the Higgs, quark, and lepton superfields are in obvious notation and the ellipsis stands for (irrelevant) higher-dimensional operators. One possible PQ-charge assignment for which the above superpotential is $U(1)_{PQ}$ invariant is given in Table I. One can also introduce additional discrete R symmetry with which the superpotential of Eq. (2) is the most generic superpotential allowed by the symmetries of the model.

To discuss the effective action at scales below f_{PQ}, let us define the axion superfield as

$$A = (\phi + ia) + \theta\tilde{a} + \theta^2 F_A,$$

where a, ϕ, and $\tilde{a}$ are the axion, saxion, and axino, respectively. It is then convenient to parametrize S_1 and S_2 as

$$S_1 = S e^{A/f_{PQ}}, \quad S_2 = S e^{-A/f_{PQ}}.$$

Note that the VEV of $S = \sqrt{S_1 S_2}$ is uniquely determined to be $\langle S\rangle = f_{PQ}$ by the superpotential (2). As will be discussed later, the VEV of $e^{\phi/f_{PQ}} = \sqrt{S_1/S_2}$ can be determined to be of order unity by SUSY-breaking effects. We then have $\langle S_1\rangle \approx \langle S_2\rangle \approx f_{PQ}$, and then f_{PQ} corresponds to the axion decay constant which would determine most of the low-energy dynamics of the axion. After integrating out the SUSY-breaking sector as well as the heavy fields in the PQ sector, the low-energy effective action includes the following Kähler potential and superpotential of the axion superfield A:

$$\begin{aligned} K_{\rm eff} = & f_{PQ}^2 \{ e^{(A+A^\dagger)/f_{PQ}} + e^{-(A+A^\dagger)/f_{PQ}} \} + \Delta K_{\rm eff} \\ W_{\rm eff} = & \mu_0 e^{2A/f_{PQ}} H_1 H_2 + \mu_i' e^{3A/f_{PQ}} L_i H_2 \\ & + e^{A/f_{PQ}} (\lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c), \end{aligned} \tag{4}$$

where $\Delta K_{\rm eff}$ is *A-dependent* loop corrections involving the SUSY-breaking effects and

$$\begin{aligned} & \mu_0 = y_0 f_{PQ}^2 / M_*, \quad \mu_i' = y_i' f_{PQ}^3 / M_*^2, \\ & \lambda_{ijk} = \gamma_{ijk} f_{PQ} / M_*, \quad \lambda'_{ijk} = \gamma'_{ijk} f_{PQ} / M_*. \end{aligned} \tag{5}$$

This shows that in our framework a weak-scale μ_0 and also small but nonvanishing R-parity-violating couplings arise as a consequence of spontaneous $U(1)_{PQ}$ breaking with $f_{PQ} \ll M_*$. In other words, the smallness of R-parity-violating couplings can be understood by means of the Frogatt-Nielsen mechanism of $U(1)_{PQ}$ [10]. Although not written explicitly, the coefficients of B-violating operators $U_i^c D_j^c D_k^c$ turn out to be small enough to avoid too rapid proton decay, *including* decays into an axino or gravitino [12].

The best lower bound on f_{PQ} from astrophysical arguments implies $f_{PQ} \gtrsim 10^9$ GeV [13]. To accommodate the LSND data, we need an axino-neutrino mixing mass of order 0.1 eV. It turns out that this value is difficult to be obtained for $f_{PQ} > 10^{10}$ GeV. We thus assume $f_{PQ} = 10^9$–10^{10} GeV with $M_* = M_{GUT}$ for which μ_0 takes a weak scale value (with appropriate value of y_0) and all R-parity-violating couplings are appropriately suppressed.

Low-energy properties of the axion superfield crucially depend on how the saxion component is stabilized. One dominant contribution to the saxion effective potential comes from $\Delta K_{\rm eff}$ which is induced mainly by threshold effects of T, T^c having an A-dependent mass $M_T = \kappa f_{PQ} e^{A/f_{PQ}}$. If $M_T \lesssim \lambda X \approx M_X$, one finds [14]

$$\Delta K_{\rm eff} \approx \frac{N_T}{16\pi^2} \frac{M_T M_T^\dagger}{\mathcal{Z}_T \mathcal{Z}_{T^c}} \ln\left(\frac{\Lambda^2 \mathcal{Z}_T \mathcal{Z}_{T^c}}{M_T M_T^\dagger} \right), \tag{6}$$

where N_T is the number of chiral superfields in T, $\mathcal{Z}_T$ is the Kähler metric of T, and Λ is a cutoff scale which is of order M_X. If $M_T \gtrsim \lambda X$, the relevant quantum correction to $K_{\rm eff}$ would be $\Delta K_{\rm eff} \sim |\lambda|^2 X X^\dagger / 16\pi^2 \mathcal{Z}_T \mathcal{Z}_{T^c}$; however, then the resulting saxion effective potential cannot stabilize ϕ at the desired VEV with $\langle e^{\phi/f_{PQ}}\rangle \approx 1$. We thus assume $M_T \lesssim \lambda X$, so $\Delta K_{\rm eff}$ is given as Eq. (6). With $\mathcal{Z}_T|_{\theta^2\bar{\theta}^2} \approx -m_{\rm soft}^2$, $\Delta K_{\rm eff}$ of Eq. (6) gives a negative-definite saxion potential

$$V_\phi^{(1)} \approx -\frac{N_T}{16\pi^2} m_{\rm soft}^2 |\kappa f_{PQ}|^2 e^{2\phi/f_{PQ}}. \tag{7}$$

There is another (positive-definite) potential from the A-dependent μ parameter:

$$V_\phi^{(2)} \approx e^{4\phi/f_{PQ}} |\mu_0|^2 (|H_1|^2 + |H_2|^2). \tag{8}$$

Then ϕ is stabilized by $V_\phi^{(1)} + V_\phi^{(2)}$ at

$$\langle e^{2\phi/f_{PQ}} \rangle \approx \frac{N_T m_{\rm soft}^2 |\kappa f_{PQ}|^2}{32\pi^2 |\mu_0|^2 (|H_1|^2 + |H_2|^2)}. \tag{9}$$

Some of our previous discussions are based on the assumption that ϕ is stabilized at $\langle e^{\phi/f_{PQ}} \rangle \approx 1$. The above expression of the saxion VEV shows that $\langle e^{\phi/f_{PQ}} \rangle \approx 1$ when κf_{PQ} is of the order of a few TeVs. This requires a rather small Yukawa coupling $\kappa \sim 10^{-6}$ which may be a consequence of some flavor symmetry. Once ϕ is stabilized by $V_\phi^{(1)} + V_\phi^{(2)}$, the resulting saxion and axino masses are given by

$$\begin{aligned} m_\phi^2 &\approx \frac{2|\mu_0|^2 (|H_1|^2 + |H_2|^2) e^{4\phi/f_{PQ}}}{f_{PQ}^2} + \Delta m_\phi^2 \\ &\approx (10\text{–}10^2 \ \text{keV})^2 + \Delta m_\phi^2, \\ m_{\tilde a} &\approx \frac{2\mu_0 H_1 H_2 e^{2\phi/f_{PQ}}}{f_{PQ}^2} + \Delta m_{\tilde a} \\ &\approx (10^{-4}\text{–}10^{-2} \ \text{eV}) + \Delta m_{\tilde a}, \end{aligned} \tag{10}$$

where the numbers in the brackets represent the gauge-mediated contributions for $f_{PQ} = 10^9$–10^{10} GeV, $\mu_0 \approx 300$ GeV, and $\langle e^{\phi/f_{PQ}} \rangle \approx 1$, and Δm_ϕ and $\Delta m_{\tilde a}$ are the supergravity-mediated contributions which will be discussed in the subsequent paragraph.

The supergravity-mediated contributions to the saxion and axino masses can be quite model dependent, in particular dependent on the couplings of light moduli in the underlying supergravity model. However, they are still generically of the order of the gravitino mass $m_{3/2}$ [15]. One model-independent supergravity-mediated contribution is from the auxiliary component u of the off-shell supergravity multiplet. In the Weyl-compensator formulation, u corresponds to the F component of the Weyl-compensator superfield:

$$\Phi = 1 + \theta^2 F_\Phi, \tag{11}$$

where the scalar component of Φ is normalized to be unity and the F component is given by [16]

$$F_\Phi = e^{K/6} \left(m_{3/2} + \frac{F_I}{3} \frac{\partial K}{\partial \Phi_I} \right), \tag{12}$$

where $F_I = -e^{-K/2} K^{IJ} \partial(e^K W^\dagger)/\partial \Phi_J^\dagger$ denotes the F component of Φ_I for the inverse Kähler metric K^{IJ} which is determined by the Kähler potential K of the underlying supergravity model. Note that Φ is defined as a dimensionless superfield, so F_Φ has mass dimension 1. The above expression shows that F_Φ is generically of order $m_{3/2}$. However, it can be significantly smaller than $m_{3/2}$ in some specific models. For instance, in the no-scale model with $K = -3 \ln(T + T^\dagger - \Phi_i \Phi_i^\dagger)$ and $\partial W / \partial T = 0$, one easily finds $F_\Phi = 0$.

The Weyl-compensator contribution to the saxion and axino masses can be easily read off from the super-Weyl-invariant supergravity action on superspace [16]:

$$-3 \int d^4\theta \, \Phi \Phi^\dagger e^{-K/3} + \left[\int d^2\theta \, \Phi^3 W + \text{H.c.} \right]. \tag{13}$$

This gives the following couplings of Φ to the axion superfield:

$$\int d^4\theta \, \Phi \Phi^\dagger K_{\rm eff} = \int d^4\theta \, \Phi \Phi^\dagger (A + A^\dagger)^2 + \cdots, \tag{14}$$

where $K_{\rm eff}$ is the effective Kähler potential in Eq. (4). It is then straightforward to see that the Weyl-compensator contributions to the saxion and axino masses are

$$\begin{aligned} \Delta m_\phi^2 &= 2|F_\Phi|^2 = \mathcal{O}(m_{3/2}^2), \\ \Delta m_{\tilde a} &= F_\Phi = \mathcal{O}(m_{3/2}). \end{aligned} \tag{15}$$

In gauge-mediated SUSY-breaking models [11], the precise value of $m_{3/2}$ depends on the details of the SUSY-breaking sector. However, most models give $m_{3/2} \gtrsim 1$ eV, implying that $m_{\tilde a}$ is dominated by a supergravity contribution. In this paper, we assume that $\Delta m_{\tilde a} \sim 1$ eV, so

$$m_{\tilde a} \approx 1 \ \text{eV}, \tag{16}$$

which would allow the axino to be a sterile neutrino for the LSND data. We note again that although it is generically of order $m_{3/2}$, $\Delta m_{\tilde a}$ can be significantly smaller than $m_{3/2}$ when the supergravity Kähler potential takes a particular form, e.g., the no-scale form [15].

Having defined our supersymmetric axion model, we discuss the full 4×4 axino-neutrino mass matrix

$$\frac{1}{2} m_{\alpha\beta} \nu_\alpha \nu_\beta, \tag{17}$$

where $\alpha, \beta = s, e, \mu, \tau$ and $\nu_s \equiv \tilde a$ with $m_{ss} = m_{\tilde a}$. The effective superpotential $W_{\rm eff}$ in Eq. (4) gives the following superpotential couplings:

$$\int d^2\theta \left[\mu_0 \left(1 + \frac{2A}{f_{PQ}} \right) H_1 H_2 + \mu_i' \left(1 + \frac{3A}{f_{PQ}} \right) L_i H_2 \right]. \tag{18}$$

We will work in the field basis in which $\mu_i' L_i H_2$ ($i = e, \mu, \tau$) in $W_{\rm eff}$ are *rotated away* by an appropriate unitary rotation of H_1 and L_i. After this unitary rotation, the above superpotential couplings are changed to

$$\int d^2\theta \left[\mu_0 \left(1 + \frac{2A}{f_{PQ}} \right) H_1 H_2 + \frac{\mu_i' A}{f_{PQ}} L_i H_2 \right], \tag{19}$$

TABLE II. Allowed regions for the LSND oscillation.

	$\|\Delta m_{41}^2\|$ (eV^2)	$\|U_{e4}\|$	$\|U_{\mu 4}\|$
R1	0.21–0.28	0.077–0.1	0.56–0.74
R2	0.88–1.1	0.11–0.13	0.15–0.2
R3	1.5–2.1	0.11–0.16	0.09–0.14
R4	5.5–7.3	0.13–0.16	0.12–0.16

leading to the axino-neutrino mass mixing

$$m_{is}=\frac{\epsilon_i\mu_0\langle H_2\rangle}{f_{PQ}}\approx 0.1\left(\frac{\epsilon_i}{10^{-5}}\right)\left(\frac{\mu_0}{600\ \text{GeV}}\right)\left(\frac{10^{10}\ \text{GeV}}{f_{PQ}}\right)\ \text{eV}, \tag{20}$$

where $\epsilon_i=\mu_i'/\mu_0$. Note that this axino-neutrino mixing *survives* under the unitary rotation eliminating $\mu_i' L_i H_2$ since L_i and H_1 have *different* $\text{U}(1)_{PQ}$ charges. This charge difference is also responsible for the suppression of R-parity-violating couplings as well as the weak-scale value of μ_0.

The 3×3 mass matrix of active neutrinos is induced by R-parity-violating couplings. At the tree level,

$$m_{ij}\approx\frac{g_a^2\langle\tilde{\nu}_i^\dagger\rangle\langle\tilde{\nu}_j^\dagger\rangle}{M_a}, \tag{21}$$

where M_a denote the gaugino masses. The sneutrino VEV $\langle\tilde{\nu}_i\rangle$ are determined by the bilinear R-parity violations in the SUSY-breaking scalar potential: $m_{L_iH_1}^2 L_i H_1^\dagger + B_i' L_i H_2$. In our model, nonzero values of $m_{L_iH_1}^2$ and B_i' at the weak scale arise through renormalization group evolution (RGE), mainly by the coupling $\lambda_{i33}' y_b$ where y_b is the b-quark Yukawa coupling [17]. Moreover, BH_1H_2 arises also through RGE which predicts a large $\tan\beta\approx 40$–60 [18]. We then find

$$m_{ij}\approx 10^{-2}t^4\left(\frac{\lambda_{i33}' y_b}{10^{-6}}\right)\left(\frac{\lambda_{j33}' y_b}{10^{-6}}\right)\ \text{eV}, \tag{22}$$

where $t=\ln(M_X/m_{\tilde{l}})/\ln(10^3)$ for the slepton mass $m_{\tilde{l}}$. Here we have taken $m_{\tilde{l}}\approx 300$ GeV and $\mu_0\approx 2m_{\tilde{l}}$ which has been suggested to be the best parameter range for correct electroweak symmetry breaking [18]. There are also bunch of loop corrections to m_{ij} from R-parity-violating couplings [19]; however, in our case they are too small to be relevant.

Let us now see how nicely all the neutrino masses and mixing parameters are fitted in our framework. The analysis of Ref. [4] leads to the four parameter regions, R1–R4 of Table II, accommodating the LSND with short-base-line results. In our model, Eqs. (16) and (20) can easily produce the LSND mass eigenvalue $m_4\approx m_{ss}=m_{\tilde{a}}\sim 1$ eV and also the LSND oscillation amplitude

$$A_{LSND}=4U_{e4}^2U_{\mu 4}^2\approx 4\left(\frac{m_{es}}{m_{ss}}\right)^2\left(\frac{m_{\mu s}}{m_{ss}}\right)^2 \tag{23}$$

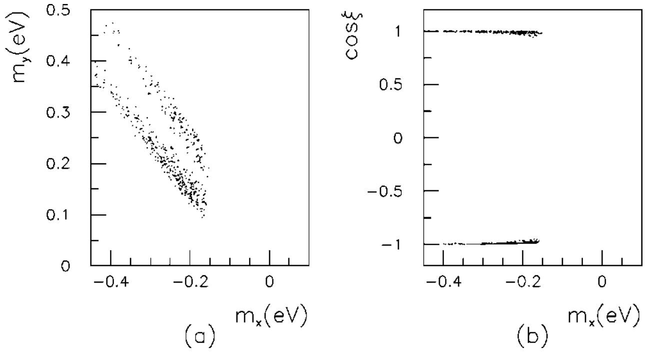

FIG. 1. Scatter plots on the (a) (m_y, m_x) plane and (b) $(\cos\xi, m_x)$ plane reproducing the correct oscillation parameters for R1.

as the four mixing elements $U_{\alpha 4}$ of the 4×4 mixing matrix U are given by $U_{i4}\approx m_{is}/m_{ss}\approx 0.1$ $(i=e,\mu,\tau)$ and $U_{s4}\approx 1$.

The masses and mixing of three active neutrinos can be easily analyzed by constructing the effective 3×3 mass matrix given by

$$m_{ij}^{\text{eff}}=m_{ij}-\frac{m_{is}m_{js}}{m_{ss}}. \tag{24}$$

Upon ignoring the small loop corrections, this mass matrix has rank 2, and can be written as

$$m_{ij}^{\text{eff}}=m_x\hat{\mathbf{x}}_i\hat{\mathbf{x}}_j+m_y\hat{\mathbf{y}}_i\hat{\mathbf{y}}_j \tag{25}$$

where $\hat{\mathbf{x}}_i$ and $\hat{\mathbf{y}}_j$ are unit vectors in the direction of m_{is} and $\langle\tilde{\nu}_j\rangle$, respectively. Remarkably, the mass scale $m_x\approx(m_{is}/m_{ss})^2 m_{ss}\sim 10^{-2}$ eV gives the right range of the atmospheric neutrino mass. Equation (22) shows that m_y is also in the range of 10^{-2} eV, so m^{eff} would be able to provide the right solar neutrino mass *unless* $\Delta m_{sol}^2\ll 10^{-4}$ eV^2. Note from Eq. (5) that the typical size of $\epsilon_i,\lambda_{ijk},\lambda_{ijk}'$ is around 10^{-6} for $f_{PQ}\approx 10^{10}$ GeV and $M_*\approx 10^{16}$ GeV.

To discuss in detail the atmospheric and solar neutrino oscillations coming from the mass matrix (25), we make an analytic diagonalization by noticing first that the eigenvector

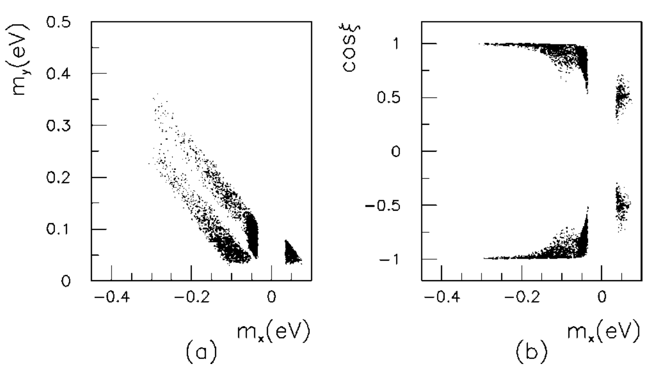

FIG. 2. Scatter plots on the (a) (m_y, m_x) plane and (b) $(\cos\xi, m_x)$ plane reproducing the correct oscillation parameters for R2.

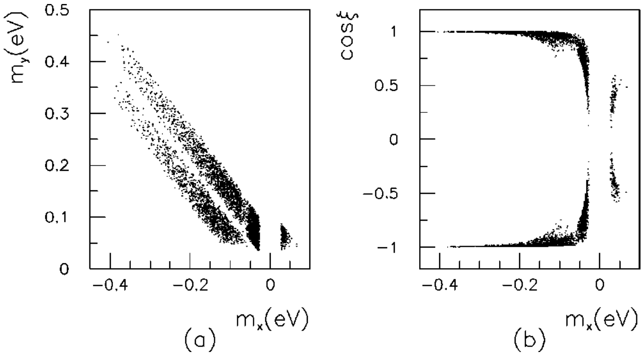

FIG. 3. Scatter plots on the (a) (m_y, m_x) plane and (b) $(\cos\xi, m_x)$ plane reproducing the correct oscillation parameters for R3.

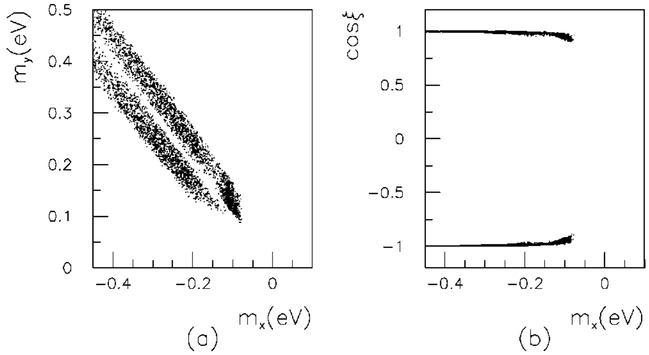

FIG. 4. Scatter plots on the (a) (m_y, m_x) plane and (b) $(\cos\xi, m_x)$ plane reproducing the correct oscillation parameters for R4.

of the massless state is given by $\hat{\mathbf{z}} \equiv \hat{\mathbf{x}} \times \hat{\mathbf{y}} / |\hat{\mathbf{x}} \times \hat{\mathbf{y}}|$. Then performing the 3×3 unitary rotation by $U_1 = (\hat{\mathbf{z}}^T, \hat{\mathbf{w}}^T, \hat{\mathbf{x}}^T)$ with $\hat{\mathbf{w}} \equiv \hat{\mathbf{x}} \times \hat{\mathbf{z}} / |\hat{\mathbf{x}} \times \hat{\mathbf{z}}|$, we get

$$U_1^T m^{\rm eff} U_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_y s_\xi^2 & m_y c_\xi s_\xi \\ 0 & m_y c_\xi s_\xi & m_x + m_y c_\xi^2 \end{pmatrix}, \qquad (26)$$

where $c_\xi \equiv \cos\xi = \hat{\mathbf{x}} \cdot \hat{\mathbf{y}}$. Note that the vector $\hat{\mathbf{y}}$ is given by $\hat{\mathbf{y}} = c_\xi \hat{\mathbf{x}} - s_\xi \hat{\mathbf{w}}$. The mass matrix (26) can be further diagonalized by the rotation $U_2 \equiv R_{23}(\theta)$ to get the mass eigenvalues

$$m_{2,3} = \frac{1}{2}\left(m_x + m_y \pm \sqrt{(m_x + m_y \cos^2 2\xi)^2 + m_y^2 \sin^2 2\xi}\right), \qquad (27)$$

where the rotation angle θ is determined by $\tan 2\theta = m_y \sin 2\xi / (m_x + m_y \cos 2\xi)$. With this procedure, the 3×3 submatrix $U_{3\times 3}$ of the full 4×4 mixing matrix U is found to be

$$U_{3\times 3} = U_1 U_2 = (\hat{\mathbf{z}}^T, \hat{\mathbf{w}}^T c_\theta - \hat{\mathbf{x}}^T s_\theta, \hat{\mathbf{w}}^T s_\theta + \hat{\mathbf{x}}^T c_\theta). \qquad (28)$$

The Super-Kamiokande data [20] combined with the CHOOZ result [21] imply that $U_{\mu 3}^2 \approx U_{\tau 3}^2 \approx 1/2$ and $U_{e3}^2 \ll 1$. The solutions to the solar neutrino problem can have either a large mixing angle (LA)

$$U_{e1}^2 \approx U_{e2}^2 \approx 1/2$$

or a small mixing angle (SA)

$$U_{e1}^2 \approx 1.$$

This specifies the first column $\hat{\mathbf{z}}^T$ of $U|_{3\times 3}$ as

$$(\text{LA}): \quad \hat{\mathbf{z}} \approx (1/\sqrt{2}, -1/2, 1/2), \qquad (\text{SA}): \quad \hat{\mathbf{z}} \approx (1,0,0)$$

up to sign ambiguities. Since $\hat{\mathbf{x}} \cdot \hat{\mathbf{z}} = 0$, the pattern $\hat{\mathbf{z}} \approx (1,0,0)$ implies $\hat{\mathbf{x}}_e \approx 0$. This leads to a too small $U_{e4} \approx m_{es}/m_{ss} \lesssim 10^{-2}$, so the SA solution is *not allowed* within our model. Among various LA solutions to the solar neutrino problem, *only* the large-angle MSW solution with $\Delta m^2_{sol} \sim 10^{-4}$ eV^2 can be naturally fitted since $m_x \approx m_y \sim 10^{-2}$ eV in our scheme. It is remarkable that $f_{PQ} \approx 10^{10}$ GeV and $M_* \approx M_{GUT}$ lead to the right size of *R*-parity violation, yielding the desired values of m_{is} and m_{ij} also for the atmospheric and solar neutrino masses.

To see the feasibility of our whole scheme, we scanned our parameter space to reproduce the allowed LSND islands R1–R4 of Table II together with the following range of atmospheric and solar neutrino parameters [1]:

$$\Delta m^2_{31} = (1-8) \times 10^{-3} \ \ \text{eV}^2,$$

$$\Delta m^2_{21} = (0.1-8) \times 10^{-4} \ \ \text{eV}^2,$$

$$\tan^2\theta_{23} = 0.33\text{–}3.8,$$

$$\tan^2\theta_{12} = 0.2\text{–}3.0,$$

$$\tan^2\theta_{13} \lesssim 0.055.$$

Our parameter space consists of $m_{ss}, m_{is}, \lambda'_{i33} y_b$ whose values are centered around 1 eV, 0.1 eV, 10^{-6}, respectively. For R1 and R4, we could find some limited parameter spaces which produce the corresponding oscillation parameters; however, they need a strong alignment between $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ and also a large cancellation between $m_{\mathbf{x}}$ and $m_{\mathbf{y}}$. On the other hand, R2 and R3 do not require any severe fine-tuning of parameters, although bimaximal mixing is obtained by some accident. We provide in Figs. 1–4 scatter plots on the planes of (m_y, m_x) and $(\cos\xi, m_x)$ for the LSND islands R1–R4.

In conclusion, we have shown that the 3+1 scheme of four-neutrino oscillations can be nicely obtained in a supersymmetric model endowed with the PQ solution to the strong *CP* problem and supersymmetry breaking mediated by gauge interactions. In this model, the axino can be as light as 1 eV, and so can play the role of a sterile neutrino. A proper axino-neutrino mixing can be induced by *R*-parity violating couplings which appear as a consequence of the spontaneous breaking of $\mathrm{U}(1)_{PQ}$. It turns out that only the large-angle MSW solution to the solar neutrino problem is allowed in our model. The weak-scale value of the Higgs μ parameter and the required size of *R*-parity violation can be understood by means of the Frogatt-Nielsen mechanism of spontaneously broken $\mathrm{U}(1)_{PQ}$.

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